Perfect square-
a quadratic formed by FOILing out an expression of the form $(x + c)^2$, where c is a constant.

Example: $(x + 9)^2 = x^2 + 18x + 81$

Example 1: Find the value of c that makes $x^2 + 14x + c$ a perfect square. c = 49

To find- divide the coefficient of the linear (x) term by two and then square the number $\frac{14}{2} \to 7^2 \to 49$

Completing the square steps:
1. make sure "a" is 1 (if not, then divide whole equation by the coefficient to get 1)
2. move the constant term to the right hand side of the equation such that you will get $x^2 + bx = c$
3. complete the square by dividing b by 2, square your answer, and add it to BOTH sides of the equation
4. Factor the left hand side of the equation (should be a perfect square)
5. Take the square root and solve the equation

$\frac{2x^2 - 4x - 16}{2} = \frac{0}{2}$

$x^2 - 2x - 8 = 0$

$x^2 - 2x - 8 = 0$ $+8 \quad +8$

$x^2 - 2x = 8$ $\frac{-2}{2} = -1$ $(-1)^2 = 1$

$x^2 - 2x + 1 = 9$

$(x-1)^2 = 9$

$\sqrt{(x-1)^2} = \sqrt{9}$

$x - 1 = \pm 3$ $+1 \quad +1$

$3 + 1 = 4$ $-3 + 1 = -2$

roots $x = 4$ and -2

Example 2: Solve $x^2 - 6x - 40 = 0$ using completing the square. $+40 \quad +40$

Step 1 - a is 1, so done.
Step 2 - move c
Step 3 - complete square- take b/2 then square add to both sides
Step 4 - factor left hand side
Step 5 - Square root and solve

$x^2 - 6x = 40$ $+9 \quad +9$

$\frac{-6}{2} = -3$ $(-3)^2 = 9$

$x^2 - 6x + 9 = 49$

$\sqrt{(x-3)^2} = \sqrt{49}$

$x - 3 = \pm 7$ $+3 \quad +3$

$x = 7 + 3$

roots $x = 10$ and -4

Example 3: Solve by completing the square $x^2 + 2x + 6 = 0$. $-6 \quad -6$

Step 1 - a is 1, so done.
Step 2 - move c
Step 3 - complete square- take b/2 then square add to both sides
Step 4 - factor left hand side
Step 5 - Square root and solve
Can't take the √ of a negative, so no real roots

$x^2 + 2x = -6$ $+1 \quad +1$

$\frac{2}{2} = 1^2 = 1$

$x^2 + 2x + 1 = -5$

$\sqrt{(x+1)^2} = \sqrt{-5}$

$x + 1 = \pm\sqrt{-5}$

Example 4: Solve $-2x^2 + 10x + 14 = 0$ by completing the square. $\frac{}{-2} \quad \frac{}{-2}$

Step 1 - since a = -2, divide everything.
Step 2 - move c
Step 3 - complete square- take b/2 then square add to both sides
Step 4 - factor left hand side
Step 5 - Square root and solve

$x^2 - 5x + 7 = 0$ $+7$

$x^2 - 5x = 7$ $+6.25 \quad +6.25$

$\frac{-5}{2} = -2.5$ $(-2.5)^2 = 6.25$

$x^2 - 5x + 6.25 = 13.25$

$\sqrt{(x-2.5)^2} = \sqrt{13.25}$

$x - 2.5 = \pm 3.6\ldots$ $+2.5 \quad +2.5$

roots $x \approx 6.1$ and -1.1

Example 5: Solve using completing the square. $x^2 - 8x + 11 = 0$ $-11 \quad -11$

Step 1 - a is 1, so done.
Step 2 - move c
Step 3 - complete square- take b/2 then square add to both sides
Step 4 - factor left hand side
Step 5 - Square root and solve

$x^2 - 8x = -11$ $+16 \quad +16$

$\frac{-8}{2} = -4$ $(-4)^2 = 16$

$x^2 - 8x + 16 = 5$

$\sqrt{(x-4)^2} = \sqrt{5}$

$x - 4 = \pm\sqrt{5}$ $+4$

roots $x = 4 \pm \sqrt{5}$